Ch. 27: Gauss’ Law

**Gauss’ Law**

\[ \Phi = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0} \]

**What in the world is this good for??**

Well, you can use it to determine unknown electric field values from a procedure much the reverse of what we used to find the flux through the spherical surface centered on the point charge.

**1) Pick a Gaussian surface reflective of the symmetry of the charge (distribution) for which you’d like to determine the electric field.**

*E.g.* For our point charge, \( Q \), case, we select a sphere.

Notice that each field line is perpendicular to the sfc.

**2) Evaluate the integral on the left-hand side.**

We always pick Gaussian surfaces such that \( \vec{E} \) is perpendicular to \( d\vec{A} \) or parallel to \( d\vec{A} \), making the integral easy to evaluate.

Since we know the electric field of a point charge is

\[ \vec{E} = \frac{kQ}{r^2} \hat{r} \]

We also know that \( \vec{E} \) is constant over the Gaussian surface.
2) Evaluate the integral on the left-hand side.

We also know that for all patches on the surface of the sphere, points in the same direction as \( \vec{E} \),

\[ \vec{E} = \frac{kQ}{r^2} \]

These two facts make the integral easy to evaluate.

The integral evaluates to\[ E(4 \pi r^2) \]

3) Evaluate the right-hand side by simply counting up the charges inside the surface (requires symmetric charge distributions)!

In the case of the point charge, we get

\[ \frac{Q}{\varepsilon_0} \]

4) Solve for the value of the electric field at the Gaussian surface.

\[ E(4 \pi r^2) = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{Q}{(4 \pi \varepsilon_0 r^2)} = \frac{kq}{r^2} \]

Same result as our definition for a point charge!

How do you pick an appropriate Gaussian surface?

Pick surface that is either parallel, perpendicular, or both to the electric field lines.

For this class, the Gaussian surfaces we use:
1) sphere – for point charges or spherical shells or balls of charge.
2) cylinder – for planes or lines of charge.
We can find the electric field at the location of the top (and bottom) of the Gaussian can above the disk.

By symmetry, we know that the electric field above and near the center of such an infinite disk must point in a direction perpendicular to the plane of the disk.

Since the top of the can is parallel to the plane of the disk and each point on the top of the can is the same perpendicular distance from the disk, the integral is easy once again.

We can divide the integral into 3 pieces: the top of the can, the bottom of the can, and the sides of the can.

Let's look at each of the above three terms separately, starting with the last one for the side of the Gaussian can.

Let's move now to the top of the can.

Sides of can: We see that along the sides of the can is perpendicular to the electric field. As a result:

If the disk is positively charged, the electric field above the disk points up, while below the disk, it points down.

We can divide the integral into 3 pieces: the top of the can, the bottom of the can, and the sides of the can.
Top of can: We see that on the top of the can is parallel to the electric field. Since all points on the top of the can are the same perpendicular distance from the disk, \( \mathbf{E} = \text{const.} \) across the top of the can.

Bottom of can: The symmetry of the problem guarantees that we get the same answer here as we did for the top of the can.

Right side: Finally, we evaluate the term on the RHS of Gauss' Law—how much charge is contained within our Gaussian can?

Finally, equate this to the result from the LHS.

\[ \mathbf{E} = \frac{\sigma}{2\varepsilon_0} \]

A rather remarkable result! The infinite disk of charge has a uniform electric field normal to its surface on both sides!
Worksheet Problem #1

Determine the electric field created by an infinite line of charge with uniform charge density $\lambda$.

Worksheet Problem #2

Determine the electric field created by a ball of charge of radius $R$ with uniform charge density $\rho$.

Worksheet Problem #3

Concept Quiz!

CAUTION
WATCH YOUR STEP

Gauss' Law