Physics 152

Lecture 17

Wednesday, February 21, 2007

Chapter 26

The Electric Field

Applying the field model to electrostatics, we can predict the behavior of charges when placed in the field. We’ll look at the fields associated with a variety of charge distributions.

\[ \mathbf{E} = \frac{kq}{r^2} \mathbf{\hat{r}} \]

- \[ |\mathbf{E}| = |k| \frac{|q|}{|r|^2} \]
- \[ [\mathbf{E}] = (\text{N m}^2/\text{C}^2) \text{ (C)}/\text{m}^2 \]
- \[ [\mathbf{E}] = \text{N/C} \]

Looking at the electric field lines gives us a way to come to understand the \(1/r^2\) nature of the expression for electrostatic force...

Let's start in 2-D

What is the density of lines passing through the blue circle?

What about the green circle?
So in flat-land, the density of lines goes down by 1/distance away from the convergence point of the lines.

In the 3-D world we live in, we replace the circles of flat-land with spheres. The density of lines passing through surrounding shells will decrease like 1/distance^2, since the surface area of a sphere is $4\pi r^2$.

So, when we bring a second charge ($q$) into the neighborhood of an existing charge, the second charge will feel a force due to the electric field of the first charge. That force is given by:

$$F = qE$$

The superposition principle applies to the electric field, too! So...

$$E_{\text{total}} = E_1 + E_2 + E_3 + \ldots$$

The proof is rather straightforward...if you believe that the electrostatic force obeys the superposition principle...

What is the Total Force on $q_2$?

1) Start by calculating the electric field of charge $q_1$ at the location of charge $q_2$.

$$E_1 = \frac{kq_1}{r_1^2}$$

2) Then examine the electric field of $q_3$ at $q_2$.

$$E_2 = \frac{kq_3}{r_2^2}$$

3) Next, use the superposition principle to carefully add together the results.

$$E_{\text{total}} = E_1 + E_2$$

(i.e., in the -x direction)
What is the Total Force on $q_2$?

4) Finally, multiply by $q_2$ to get the force...

(i.e., the electric force at the location of $q_2$ points in the positive $x$ direction)

$q_1 = -1 \mu C$  \hspace{1cm} $q_2 = -2 \mu C$  \hspace{1cm} $q_3 = +1 \mu C$  

$r_1 = 1 \text{ m}$  \hspace{1cm} $r_2 = 2 \text{ m}$

$$F_{tot} = q_2 E_{tot} = \left(2 \times 10^{-6}\right) \left(11.25 \times 10^3 \text{ N/C}\right) \hat{i} = 2.25 \times 10^{-2} \text{ N} \hat{i}$$

What is the Electric field on the $y$-axis?

A vector that points from the negative to the positive charge and has magnitude:

$$p = qd = q(2x)$$

In this particular case

$$\vec{p} = -2qxi$$

Using our expression for the electric dipole moment, the field then becomes (for positions along the $x$- and $y$- axes much greater than the separation distance of the two charges):

$$\mathbf{E} = \frac{-k\rho}{y^3} \hat{i}$$

parallel

$$\mathbf{E} = \frac{2k\rho}{x^3} \hat{j}$$

perpendicular
What if the charge is NOT a point charge, but rather a distribution of charge in space?

While that certainly complicates matters a bit, we can still attack problems that involve simple charge distribution geometry.

For charge distributed in a volume (e.g., a ball of charge), we define the charge per unit volume:

$$\rho = \frac{Q}{V}$$

Note: as was the case with gravity of planetary masses, spherical charge distributions produce electric fields that look the same as those associated with a point charge of the same total value located at the center of the sphere!

For charge distributed over a surface (e.g., a plane of charge), we define the charge per unit area:

$$\sigma = \frac{Q}{A}$$

For charge distributed along a line (e.g., a line of charge), we define the charge per unit length:

$$\lambda = \frac{Q}{L}$$

We can break our charge distribution up into a bunch of little point charges, dq, and then add up the results using the superposition principle to get the total electric field.

In the limit that the dq’s become infinitesimal, our result becomes

$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

The nature of the dq’s depends on the nature of the charge distribution, and so too does the integral itself.

If the charge is distributed over a volume of space, for example, our integral becomes a 3D or volume integral, and we’d use
Find the electric field a distance $s$ away from the center of and along the perpendicular bisector of a uniform line of charge of length $L$ with charge per unit length $\lambda$. 

Worksheet Problem #3

Concept Quiz!

Distributed Charge Fields

CAUTION

WATCH YOUR STEP