Physics 111

Lecture 24

Thursday, December 2, 2004

- Ch 17: Latent Heat
  Ideal Gas Law
  Kinetic Theory
  Stress
  Strain
The Physics 111 Help Session

For the rest of the semester

- Sunday, Dec. 5, 6:30 - 8 pm in CCLIR 468
- Thursday, Dec. 9, 3 - 5pm and 7 - 9pm
Announcements

Don’t forget to prepare for a short quiz.

Last Lab Meets This Week
Announcements

Monday, Dec. 13, 2004
10:30 am – 12:30 pm
everything

• Hint: Be able to do the homework (graded AND recommended) and you’ll do fine on the exam!

• You may bring one 8.5”X11” crib sheet (hand-written on both sides), a pencil or pen, and a scientific calculator with you.

• I will put any constants and mathematical formulas that you might need on a single page attached to the back of the exam.
The review of your chemistry class continues. In this chapter, we look at phase transitions, the kinetic theory of gases, and solid deformations.
When a substance changes phase, extra energy is either released or required.

We classify matter as existing in one of 3 phases:

- solid
- liquid
- gas
If we start out with our substance in a gaseous phase, then how do we get it to become a liquid?

Think of the result of a lower temperature in terms of the molecular argument about the speed of the molecules...

Cool It!

Energy is released as a gas becomes a liquid!
In order to get a substance to change phase from a liquid to a gas, we must...

We have to add energy to vaporize a liquid.

The amount of energy we must add to get a substance to change from a *liquid to a gas* is the same as the amount of energy that is released when the same substance changes phase from a *gas to a liquid*…and is known as

\[ L_v \]

the latent heat of vaporization
On what property of our substance could the latent heat (that is, the amount of heat necessary to melt our substance from a solid to a liquid of to vaporize our liquid to a gas) possibly depend?

How much of the substance we’ve got.

Clearly, the more of it I’ve got, the more heat I’m going to have to transfer to that substance to get a given temperature change.

We specify this quantity with mass \( m \).
Should this graph pass through the origin?

That is, how much heat must be transferred to get the phase to change for an amount $m$ of the substance?

The slope of this line is the *Latent heat* ($L$) of either fusion of vaporization, depending upon the phase change we’re examining.
Should this graph pass through the origin?

That is, how much heat must be transferred to get the phase to change for an amount $m$ of the substance?

$Q = mL$
A 100.0 g ice cube is initially at $T_i = -30.0^\circ C$. The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at $T_f = 120^\circ C$. How much total heat was transferred to the ice?

Use the following information:

- $c_{H_2O}(s) = 2090 \text{ J/kg/}^\circ \text{C}$
- $L_f = 3.33 \times 10^5 \text{ J/kg}$
- $c_{H_2O}(l) = 4190 \text{ J/kg/}^\circ \text{C}$
- $L_v = 2.26 \times 10^6 \text{ J/kg}$
- $c_{H_2O}(g) = 2010 \text{ J/kg/}^\circ \text{C}$
A 100.0 g ice cube is initially at $T_i = -30.0^\circ$C. The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at $T_f = 120^\circ$C. How much total heat was transferred to the ice?

First, we must warm the ice to the temperature at which it melts ($0^\circ$C). This requires heat:

$$Q_1 = m_{H_2O} c_{H_2O} (s) \Delta T_{ice}$$

$$Q_1 = (0.1 \text{ kg})(2090 \frac{\text{J}}{\text{kg} \cdot ^\circ\text{C}})(0^\circ \text{C} - (-30^\circ \text{C}))$$

$$Q_1 = 6,270 \text{ J}$$
Next, we must melt the ice. This requires heat:

\[ Q_2 = m_{H_2O} L_f \text{ H}_2\text{O} \]

\[ Q_2 = (0.1 \text{ kg})(3.33 \times 10^5 \frac{\text{ J}}{\text{ kg}}) \]

\[ Q_2 = 33,300 \text{ J} \]
A 100.0 g ice cube is initially at \( T_i = -30.0^\circ C \). The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at \( T_f = 120^\circ C \). How much total heat was transferred to the ice?

Then, we must warm the water to the temperature at which it boils (100\(^\circ\)C). This requires heat:

\[
Q_3 = m_{H_2O}c_{H_2O}(l)\Delta T_{H_2O(l)}
\]

\[
Q_3 = (0.1 \text{ kg})(4190 \text{ J/kg}^\circ C)(100^\circ C - 0^\circ C)
\]

\[
Q_3 = 41,900 \text{ J}
\]
A 100.0 g ice cube is initially at $T_i = -30.0^\circ C$. The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at $T_f = 120^\circ C$. How much total heat was transferred to the ice?

Next, we must change the liquid water into steam. This requires heat:

$$Q_4 = m_{H_2O}L_{vH_2O}$$

$$Q_4 = (0.1 \text{ kg})(2.26 \times 10^6 \text{ J/kg})$$

$$Q_4 = 226,000 \text{ J}$$
A 100.0 g ice cube is initially at $T_i = -30.0^\circ C$. The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at $T_f = 120^\circ C$. How much total heat was transferred to the ice?

Then, we must warm the steam to the temperature of 120.0\(^\circ\)C. This requires heat:

$$Q_5 = m_{H_2O}c_{H_2O}(g)\Delta T_{H_2O(g)}$$

$$Q_5 = (0.1 \text{ kg})(2010 \frac{\text{J}}{\text{kg}^\circ \text{C}})(120^\circ \text{C} - 100^\circ \text{C})$$

$$Q_5 = 4,020 \text{ J}$$
A 100.0 g ice cube is initially at $T_i = -30.0^\circ C$. The ice is placed in a sealed container over a heat source. Heat is transferred to the ice until it becomes steam at $T_f = 120^\circ C$. How much total heat was transferred to the ice?

The total amount of heat added is then just the sum:

$$Q_{tot} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_{tot} = (6,270 + 33,300 + 41,900 + 226,000 + 4,020) J$$

$$Q_{tot} = 311,490 \ J$$
Calorimetry Problems

Tips for solving such problems:

1) Make sure your units are consistent!

2) Only use the specific heat formula for temperature changes without phase changes.

\[ Q = mc\Delta T \]
Calorimetry Problems

Tips for solving such problems:

3) Only use the latent heat formula when phase changes occur!

\[ Q = mL \]

4) Set heat loss = heat gain and use ONLY positive values of \( \Delta T \)!!
A property related to the molecule itself, the atomic weight is related to the molar mass.

For example, oxygen has an atomic weight of 16. This means that 1 mole of atomic oxygen will have a mass of 16 grams.

1 mole of ANY substance contains Avagadro's number of molecules

\[ N_A = 6.022 \times 10^{23} \text{ molecules} \]
Most gases with which we deal on a day-to-day basis behave as ideal gases.

Ideal gases obey the *ideal gas law*, a physical law that relates the *pressure* of a gas, the *number* of molecules in the gas, the *volume* of the gas, and its *temperature*.
We can reason out this relationship fairly easily. Keeping everything else constant...

What happens to the temperature of a gas if we increase its pressure?

Pressure is proportional to temperature.
We can reason out this relationship fairly easily. Keeping everything else constant...

What happens to the pressure of a gas if we increase the number of molecules for fixed $V$ and $T$?

Pressure is proportional to the number of molecules.

$P \propto N$
The Ideal Gas Law

We can reason out this relationship fairly easily. Keeping everything else constant...

What happens to the volume of a gas if we increase the number of molecules at fixed $P$ and $T$?

**Volume** is proportional to $N$. $V \propto N$
We can reason out this relationship fairly easily. Keeping everything else constant...

What happens to the pressure of a gas if we increase its volume?

So pressure is inversely related to volume.

\[ P \propto \frac{1}{V} \]
Summarizing, we have:

\[ P \propto T \quad P \propto N \quad P \propto \frac{1}{V} \]

Therefore, we can confidently state that

\[ P = k_B \frac{NT}{V} \]

where \( k_B \) is the proportionality constant, \textit{Boltzman’s constant}. 

\( k_B = 1.38 \times 10^{-23} \text{ J} / \text{K} \)
The Ideal Gas Law

Summarizing, we have:

\[ P \propto T \quad P \propto N \]

More typically, the ideal gas law is written

\[ PV = Nk_B T \]
It is also often convenient to write the ideal gas law using the *number of moles* rather than the *number of molecules*. In this case, we have

\[ PV = \frac{N_A}{N_A} N k_B T = \frac{N}{N_A} (N_A k_B) T = nRT \]
So a common, but equivalent form of the ideal gas law, is:

\[ PV = nRT \]

where \( n \) is the number of moles of gas \( \left( \frac{N}{N_A} \right) \) and \( R \) is the ideal (universal) gas constant

\[ R = 8.314 \text{ J/mole/K} \]

\[ = 0.08206 \text{ L atm/mole/K} \]
What volume is occupied by 1 mole of gas at temperature 0°C and pressure (1 atm)?

1) 0
2) $3.8 \times 10^{-21}$ L
3) 22.4 L
4) 45 L
5) 2270 L
6) None of the above
What volume is occupied by 1 mole of gas at temperature 0°C and pressure (1 atm)?

\[ PV = nRT \]

\[
(1 \text{ atm})V = (1 \text{ mole})(0.0821 \frac{\text{L atm}}{\text{mole K}})(273 \text{ K})
\]

\[ V = 22.4 \text{ L} \]
How many molecules will you find in 1 cm$^3$ of gas at temperature 0$^\circ$C and pressure of 1 atm?

1) $4.52 \times 10^{-12}$
2) $4.46 \times 10^{-5}$
3) $2.65 \times 10^{14}$
4) $2.68 \times 10^{16}$
5) $2.68 \times 10^{19}$
6) None of the above

Worksheet #3
How many molecules will you find in 1 cm³ of gas at temperature 0°C and pressure of 1 atm?

\[ PV = Nk_B T \]

\[
(101.3 \times 10^3 \text{ Pa})(10^{-6} \text{ m}^3) = N (1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(273 \text{ K})
\]

\[ N = 2.68 \times 10^{19} \text{ molecules} \]
I’ve got a box full of molecules. They’re bouncing around in my box in all different directions, at a variety of speeds.
What happens when a molecule runs into a wall of my box?

*It exerts a force on that wall.* Can we figure out the magnitude of this force?
Let’s simplify our problem by examining a 1-D gas instead of a 3-D gas. Assume our molecules only move in the $x$-direction, and that all collisions they make with the walls are perfectly elastic.
What is the magnitude of the force that has been applied by the right wall to our molecule?

\[ F = m \frac{\Delta v}{\Delta t} = \frac{2mv_{xi}}{\Delta t} \]
How much time do we have to wait between applications of this force on the same molecule by the same wall?

\[ \Delta t = \frac{2d}{v_{xi}} \]
We can now plug our solution for $\Delta t$ into our expression for force and get

$$F = \frac{2m v_{xi}}{\Delta t} = \frac{2m v_{xi}}{2d / v_{xi}} = \frac{m v_{xi}^2}{d}$$

This is the force of the wall on a single molecule.

By Newton’s 3rd Law, this must also equal the force of a single molecule on the wall.
So, now let’s look at the **total force** on the wall, which will be the **sum of the forces** exerted by each molecule:

\[
F_{\text{tot}} = \frac{m}{d} \left( v_{x1}^2 + v_{x2}^2 + \ldots + v_{xN}^2 \right)
\]

We note that the **average square velocity** of our molecules is simply

\[
< v_x^2 > = \frac{\left( v_{x1}^2 + v_{x2}^2 + \ldots + v_{xN}^2 \right)}{N}
\]
So we can now rewrite our force in terms of this average velocity squared quantity:

\[ F_{tot} = \frac{m}{d} N < v_x^2 > \]

Now, we expand our analysis to a 3-D gas. In a 3-D gas, no direction is special. This implies that the mean square velocity in the \( x \)-direction equals that in the \( y \) and \( z \) directions as well.

\[ < v_x^2 > = < v_y^2 > = < v_z^2 > \]
By the Pythagorean theorem

\[
\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle
\]

where \( v \) is the total velocity of the molecule (i.e., the vector sum of \( v_x, v_y, \) and \( v_z \)).

But since the average speed in each direction is the same

\[
\langle v^2 \rangle = 3 \langle v_x^2 \rangle
\]
So we can now rewrite our force in terms of this 3-D average velocity squared:

\[ F_{\text{tot}} = \frac{m}{d} N < v^2 > \]

One last step... now let's find the pressure exerted on a wall. Remember, pressure is force divided by area.

\[ P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3} \left( \frac{N}{d^3} m < v^2 > \right) \]
Manipulating this just a bit, we get:

\[ P = \frac{2}{3} \left( \frac{N}{V} \left( \frac{1}{2}m < v^2 > \right) \right) \]

So the pressure is proportional to the number of molecules per unit volume times the average kinetic energy of a molecule!
We have successfully explained a macroscopic quantity (Pressure) in terms of microscopic quantities (the kinetic energy of each molecule)!

This is quite a remarkable achievement, and it tells us that links exist between the atomic world and the large-scale world in which we spend most of our time.
Let’s rewrite our equation for pressure in a form that resembles the ideal gas law:

\[
P = \frac{2}{3} \left( \frac{N}{V} \left( \frac{1}{2} m < v^2 > \right) \right)
\]

Recall, the ideal gas law says:

\[
P V = N k_B T
\]
Equating the right-hand sides of these equations we get

\[ Nk_B T = \frac{2}{3} N \left( \frac{1}{2} m < v^2 > \right) \]

We now see directly the relationship between Temperature and molecular kinetic energy.
An ideal gas has an initial temperature of 20\(^0\)C. The gas is heated, and the kinetic energy is observed to increase by a factor of 2. What is the final temperature of the gas?

1) 20\(^0\)C  
2) 40\(^0\)C  
3) 80\(^0\)C  
4) 313\(^0\)C  
5) 586\(^0\)C  
6) None of the above