Physics 111

Lecture 14

Tuesday, October 12, 2004

• Ch 7: Work
  Hooke’s Law

• Ch 8: Potential Energy
  - gravity
  - spring
  Conservative Forces
Help this week:
Wednesday, 8 - 9 pm in NSC 118/119
Sunday, 6:30 - 8 pm in CCLIR 468
Don’t forget to read over the lab and prepare for the short quiz.
Exam #2

Tuesday, Oct. 19, 2004
11:50 am – 1:05 pm
Ch. 5, 6, 9.1 - 9.4

- **Hint:** Be able to *do the homework* (graded AND *recommended*) and you’ll do fine on the exam!

- You may bring one 3”X5” index card (*handwritten on both sides*), a *pencil or pen*, and a *scientific calculator* with you.

- I will put any constants and mathematical formulas that you might need on a single page attached to the back of the exam.
Exam #2

Tuesday, Oct. 19, 2004
11:50 am – 1:05 pm
Ch. 5, 6, 9.1 - 9.4

Format:

• **Three sections** (like homework). You do any two of the three.

• **One essay question.** Required for all.

• **One section of multiple choice.** 5 questions roughly evenly divided. Required for all.
We’ve looked at work for constant forces. What happens if the force is not constant.

Given any non-constant force as a function of position:

We can treat the function as constant if we look at a small enough interval $\Delta x$!

$$\Delta W = F_x(x_i)\Delta x$$
So the total work done over some displacement

$$\Delta x = x_f - x_i$$

is simply the sum of all the $\Delta W$ of each little interval.

$$\mathcal{W} = \lim_{\Delta x \to 0} \sum_{i=1}^{\infty} F_x (x_i) \Delta x$$
Let’s look at a specific example system in which the applied force is NOT constant.

Sir Robert Hooke unlocked the secret of the spring...

A spring resting in its natural state, with a length L exerts no horizontal force on anything!
However, if we compress or stretch the spring by some amount $\Delta x$, then the spring is observed to exert a force in the opposite direction.

Hook discovered this force could be modeled by the mathematical expression

$$\vec{F} = -k\Delta \vec{x}$$

Notice that this force operates along a linear line!
Which means that if we looked at the plot of \textit{Force} versus compression/stretching $\Delta x$...

Slope of this line is $-k$, where $k$ is the spring constant.
A spring has a relaxed length of 20.0 cm. With one end attached to a wall, the spring is found to stretch to a length of 22.0 cm with a pulling force of 100 N exerted on the other end. Determine the spring constant.

Math: Newton’s 2nd Law

\[
\alpha = 0 = \frac{\vec{F}_{\text{net}}}{m} = \frac{\vec{F}_{\text{me}} + \vec{F}_{\text{spring}}}{m}
\]

\[
\vec{F}_{\text{me}} = -\vec{F}_{\text{spring}} = -(-k)(\Delta x)
\]

\[
k = \frac{|\vec{F}_{\text{me}}|}{|\Delta x|} = \frac{100\text{N}}{+0.02\text{m}} = 5000 \frac{\text{N}}{\text{m}}
\]
The same spring is now used in a tug-of-war between two people. Each person pulls on his end of the spring with a force of 100 N. How long is the spring while it’s being pulled.

1) 20 cm
2) 22 cm
3) 24 cm
4) 40 cm
5) 44 cm
6) 48 cm

Remember the problem with the horses, the rope, and the tree?
If we look at the work done by the spring force when compressing the spring through a displacement \((-x_1)\)…

\[
W = F \Delta s = \overline{F}(-x_1) = \frac{1}{2} (F_1 - 0)(-x_1)
\]

\[
W = \frac{1}{2} (-k)(-x_1)(-x_1) = -\frac{1}{2} kx^2
\]

Note: The work done is just the area under the curve!
If a spring with a block on the end of it begins in a compressed state with $x = -x_1$. How much work is done by the spring after the mass is released as the mass reaches $x = 0$?
If we look at the work done by the spring force when uncompressing through a displacement \((+x_1)\),

\[
\mathcal{W} = F \Delta s = \overline{F}(+x_1) = \frac{1}{2}(F_1 - 0)(+x_1)
\]

Note: The work done is just the area under the curve!
Let’s think a little more about our gravity/bucket example from last time. We saw that a man raising a 5 kg bucket through 0.5 m at constant velocity does an amount of work

\[ W = 25 \text{ J} \]

But if the **NET WORK** done on the bucket equals the change in **kinetic energy** of the bucket, then

\[ W_{net} = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = 0 \]

What’s the deal?
Solution: Although the man does 24.5 J of work lifting the bucket, the force of gravity (acting in the opposite direction) does –24.5 J of work.

\[ W_{\text{net}} = W_{\text{man}} + W_{g} = 24.5 \text{ J} - 24.5 \text{ J} = 0 \]

The key to unlocking this conundrum was that we need to calculate the NET WORK DONE ON the bucket by ALL the forces acting!!! (Not that done by the man or by gravity alone!)
Work! \[ W = \pm |\vec{F}_s| |\Delta s| \]

$|\Delta s|$ is the magnitude of the displacement as measured along the path that the object actually travels (i.e., along its trajectory), where $\Delta s$ is small or always in the same direction.

$|F_s|$ is the component of the force in the direction of motion.

+ sign if $\vec{F}_s$ and $\Delta s$ are in the same direction.
– sign if $\vec{F}_s$ and $\Delta s$ are in opposite directions.
We’ve now looked at how forces, constant and variable, do work. But what do we mean by negative work? When we raise a bucket and gravity does negative work, to where does the energy disappear? Or has it?
Let’s think a bit more about the bucket problem.

• We certainly changed the position of the bucket relative to the ground by lifting it. If we let go, it would fall back to the ground.

• The higher we raise the bucket, the faster its speed when it does hit the ground.

Ladies and Gentlemen….INTRODUCING:

Potential Energy
Because we are in the space strongly influenced by the Earth’s gravity, we call this the **gravitational potential energy**.

Consider the force that the Earth’s gravity exerts on the 5-kg bucket as we move it from the ground to a height of 0.5 m at constant velocity.

\[
W = m \cdot g = (5 \text{ kg})(-9.8 \text{ m/s}^2) = -49 \text{ N}
\]

\[
W = F_s \Delta s = W(y_f - y_i) = (-49 \text{ N})(0.5 \text{ m} - 0.0 \text{ m}) = -24.5 \text{ J}
\]

We define **gravitational potential energy** (near Earth):

\[
U_g = m |g| y
\]
Potential Energy

Units!

\[ U_g = m \cdot g \cdot y \]

\[ [U] = [m][g][y] \]

\[ [U] = \text{kg (m/s}^2\text{)} \cdot \text{m} \]

\[ [U] = \text{N m} = \text{J} \]

YAY!!! It has the units of ENERGY!!!
Now we can compute the work done by gravity on the 5-kg bucket as we raise it from the ground to 0.5 m above the ground.

\[ W_g = -\Delta U = U_i - U_f \]

\[ U_f = m \, g \, y_f = (5 \text{ kg})(9.8 \text{ m/s}^2)(0.5 \text{ m}) = 24.5 \text{ J} \]

\[ U_i = m \, g \, y_i = (5 \text{ kg})(9.8 \text{ m/s}^2)(0.0 \text{ m}) = 0 \text{ J} \]

\[ W_g = 0 - 24.5 \text{ J} = -24.5 \text{ J} \]

Thus, we get the same answer from Potential Energy as we got with Force Balance.
• We are free to define the origin of our y-coordinate at any point above the Earth’s surface (or below it) that we find convenient -- the work done (and change in potential energy) only depends upon the CHANGE IN HEIGHT!

• Potential Energy, like kinetic energy, is a scalar.

• Our definition of gravitational potential energy can only be applied near the Earth’s surface.
We carry two marbles, one with twice the mass of the other, to the top of the same building. How does the change in gravitational potential energy of the less massive marble compare to that of the more massive marble?

1) they’re the same.
2) it’s twice as much.
3) it’s half as much.
4) it’s four times as much.
5) impossible to determine.
So...WHY DO we call it POTENTIAL energy?

If this guy holds the 5 kg bucket still 0.5 m off the ground, what is the bucket’s Kinetic energy? Potential energy?

What happens if he drops the bucket?

What are the kinetic and potential energies when the bucket hits the ground?
What do we notice about the initial potential energy and the final kinetic energy the bucket?

\[ U_i = K_f \]

Provided we define \( y = 0 \) at the ground.

We’re going to generalize now, but you should be able to prove this to yourself through a few practice problems.

\[ K_i + U_i = K_f + U_f \]

We call \( K + U \) the **TOTAL MECHANICAL ENERGY**.
The total mechanical energy of any \textit{isolated} system of objects remains constant if the objects interact only through \textit{conservative forces}. 

\[ K + U = \text{constant!} \]
We’ve got a couple of terms to define here...

• An “isolated” system implies that no energy is being added or removed from the system.

• Next, we need to come to an understanding of conservative forces...

NOT THOSE FORCES!!
The work done on a particle by a conservative force is independent of how the particle moves from one point to another.

For a conservative force, it doesn’t matter how I get from A to B, the net work done is the same on any path!

In general, the work done by a conservative force is given by

\[ W_c = -\Delta U \]
• A **force** is conservative if the total **work** it does on a particle is **zero** when the particle moves around any **closed path**, ending up exactly where it started out.

• If the ball ends up exactly back where it started under the influence of a **conservative force**, the total **work** done by that force is **ZERO**!
How much work does the gravitational force do on the tennis ball as I toss it up and it falls back to my hand?

The force of gravity is a CONSERVATIVE FORCE.

We will soon see that there are OTHER conservative forces besides gravity.