Ch 9: Momentum

- Conservation of Momentum

Ch 7: Work
- Kinetic Energy

Don’t forget to read over the lab and prepare for the short quiz.

In fact, we can relate force to momentum changes directly using Newton’s 2nd Law:

\[ \mathbf{F}_{\text{net}} = m \mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} \]

This form is completely general, as it allows for changes in mass as well as velocity with time.

But that’s just impulse. So we now have a relationship between momentum and impulse!

We define impulse:

\[ I \equiv \langle \mathbf{F}_{\text{net}} \rangle \Delta t = \Delta \mathbf{p} \]
Worksheet #1

**Impulse**

A 0.25 kg baseball arrives at the plate with a speed of 50 m/s. Barry Bonds swings and the ball leaves his bat at 40 m/s headed directly back toward the mound along the same path on which it arrived. Assuming that the ball is in contact with the bat for $10^{-3}$ s, what is the magnitude of the average force the bat exerts on the ball? Ignore gravity in this problem.

The total momentum of an isolated system of objects is **conserved**, regardless of the nature of the force between the objects in that system.

Here, we speak of the linear momentum of the system.

Thus, if our system consists of two objects...

The total initial momentum of the system is

$$\mathbf{p}_{\text{sys}i} = \mathbf{p}_1 + \mathbf{p}_2 = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

After these two balls hit, the situation changes...

The total final momentum of the system is

$$\mathbf{p}_{\text{sys}f} = \mathbf{p}_1 + \mathbf{p}_2 = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2$$

...collisions

• NOTE: our formulation above is derived for a system of two **POINT MASS** objects.

• With no external forces acting upon these objects, when the two objects collide, the total linear momentum before and after the collision are the same.
Notice that this statement can also be derived through Newton’s Laws.

Let's say the collision lasts for a time $\Delta t$.

FBDs:

\[
\begin{align*}
\vec{F}_{2\to1} & \quad m_1 \\
\vec{F}_{1\to2} & \quad m_2
\end{align*}
\]

Newton’s 3rd Law:

\[
\begin{align*}
\vec{F}_{2\to1} & = -\vec{F}_{1\to2} \\
\Delta \vec{p}_1 & = -\Delta \vec{p}_2
\end{align*}
\]

Or stated slightly differently:

\[
\begin{align*}
\vec{p}_{1,f} - \vec{p}_{1,i} & = -(\vec{p}_{2,f} - \vec{p}_{2,i}) \\
\vec{p}_{1,f} + \vec{p}_{2,f} & = -(\vec{p}_{1,i} + \vec{p}_{2,i})
\end{align*}
\]

\[
\vec{p}_{tot,f} = -(\vec{p}_{tot,i})
\]

**Concept Quiz!**

**Worksheet #2**

**Chapter 7**

Work & Kinetic Energy

After developing an understanding of motion using velocity, acceleration, forces, momentum, and impulse (all vector quantities), we’re now going to develop scalar tools that will help make some problems much simpler!
We come to physics with at least a well-defined notion of what “work” is. However, in physics, the term has a VERY SPECIFIC meaning.

**Work is the mechanical transfer of energy between a system and its environment by pushes and pulls (forces) at the boundary where the forces are applied.**

Let’s try to provide a context for understanding why “work” is such an important concept in physics.

Describe the following demonstrations:

1) Dropped ball  
   Worksheet #3

2) Tugged cart  
   Worksheet #4

Remember to identify the “system” and forces.

Mathematically, this statement of work translates to

\[ W = F_s \Delta s \]

Where \( \Delta s \) is the distance the object travels as measured along the path it follows (i.e., its trajectory) and \( F_s \) is the component of the force in the direction of motion.

This statement is correct so long as the applied force \( F_s \) is a constant.
We’re about to relate work to a new quantity by using an old expression...

\[ s_f - s_i = v_i \Delta t + \frac{1}{2} a(\Delta t)^2 \]

Now solve for \( \Delta t \)

\[ (\Delta t) = \frac{v_f - v_i}{a} \]

Plug back in above...

\[ s_f - s_i = v_i \left( \frac{v_f - v_i}{a} \right) + \frac{1}{2} a \left( \frac{v_f - v_i}{a} \right)^2 \]

This last quantity looks something like work, right? All we need to do is multiply both sides by mass, and we have force times distance!

Each term on the right-hand-side of this equation is called a "kinetic energy" term.

And now we can rewrite work as a change in kinetic energy:

\[ W_{\text{net}} = \Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \]

That's good! It has the same units as work! WHEW!

Although we've derived this relationship between work and kinetic energy for the special case of a constant applied force (i.e., constant acceleration), it turns out to be generally true, regardless of whether or not the force is constant!

The NET WORK done on an object is equal to its change in KINETIC ENERGY.
Let’s think about this a little further…
We saw that a man raising a 5 kg bucket through 0.5 m at constant velocity does an amount of work
\[ W = 25 \text{ J} \]
But if the NET WORK done on the bucket equals the change in kinetic energy of the bucket, then
\[ W_{net} = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2) = 0 \]
What’s the deal? Worksheet #5

\[ W = \pm |\vec{F}_s| |\Delta s| \]
\(|\Delta s|\) is the magnitude of the displacement as measured along the path that the object actually travels (i.e., along its trajectory), where \(|\Delta s|\) is small or always in the same direction.
\(|\vec{F}_s|\) is the component of the force in the direction of motion.
+ sign if \(\vec{F}_s\) and \(|\Delta s|\) are in the same direction.
– sign if \(\vec{F}_s\) and \(|\Delta s|\) are in opposite directions.

Worksheet #6

Ch 9: Momentum and Collisions

Concept Quiz!