Lecture 12

Tuesday, October 05, 2004

• Ch 6: Last Example
• Ch 9: Impulse
  Momentum

Ch 6: Applying Newton’s Laws

Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string passing over a frictionless pulley. The inclines are also frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.

Ch 9: Momentum and Collisions

Linear Momentum & Collisions

In the study of systems with multiple objects, particularly systems in which these objects interact with one another through collisions, it is useful to take advantage of the concept of momentum. We’ll see that this new quantity is conserved in isolated systems, allowing us to analyze some rather complicated interactions.
We've applied Newton's Laws to better understand the behavior of systems
- single object
- multiple object
- remote interactions

Now, let's examine interactions between objects through another approach:

What constitutes an impulse is probably best illustrated through a simple demonstration. When I drop the ball, the force of gravity exerts an impulse on the falling ball. When the ball hits the floor, a second force also exerts an impulse: the contact force of the floor.

For a constant force, as with the falling ball, it's easy!

\[ \vec{I} = \vec{F} \Delta t \]

Notice graphically that the impulse is the “area” under the curve of a force vs time graph. (Just like distance was the “area” under a velocity vs time graph!)

So, what about non-constant forces?

Let’s assume that during the bounce, the floor exerts a force on the ball as given at left. The answer is still just the area under the curve.

\[ \bar{F}_{\text{floor}} = \frac{F_{\text{max}} + F_{\text{min}}}{2} \Delta t = \langle \vec{F} \rangle \Delta t \]

\[ \bar{I} = \frac{1}{2} (F_{\text{max}} + F_{\text{min}}) \Delta t = \langle \vec{F} \rangle \Delta t \]

Note: The force of gravity results in an impulse at the same time.
A 0.200-kg rubber ball is released from a height of 2.00 m above the floor. The ball falls, bounces, and rebounds. How high does the ball make it on the rebound if the force function is given by the graph below?

**Problem Sheet #2**

We can always find the a constant $F$ such that the impulse delivered is the same as that delivered by a non-constant force.

The linear momentum of an object of mass $m$ moving with velocity $v$ is defined as the product of the mass and the velocity:

$$\vec{p} = m\vec{v}$$

**Units:**

$$[\vec{p}] = \text{kg \ (m/s)}$$

OR

$$[\vec{p}] = \text{N \ s}$$

The units suggest a relationship between force and momentum.
What happens when we apply a force to an object?

- It accelerates.
- Its velocity changes.
- Its momentum changes.
- The force imparts momentum.

In fact, we can relate force to momentum changes directly using Newton’s 2nd Law:

\[ F_{\text{net}} = \frac{\Delta p}{\Delta t} \]

This form is completely general, as it allows for changes in mass as well as velocity with time.

But that’s just impulse. So we now have a relationship between momentum and impulse!

We define impulse:

\[ I \equiv \left( F_{\text{net}} \right) \Delta t = \Delta p \]

Impulse-Momentum Theorem

By how much will the momentum change?

That depends upon the amount of time over which the force is applied to the object.

Here we’ll derive the case for a constant force.

\[ \vec{v}_f = \vec{v}_i + \vec{a}(\Delta t) \]

\[ m\vec{v}_f = m\vec{v}_i + m\vec{a}(\Delta t) \quad m\vec{v}_f - m\vec{v}_i = \vec{F}_{\text{net}} \Delta t \]

A 0.25 kg baseball arrives at the plate with a speed of 50 m/s. Barry Bonds swings and the ball leaves his bat at 40 m/s headed directly back toward the mound along the same path on which it arrived. Assuming that the ball is in contact with the bat for 10^-3 s, what is the magnitude of the average force the bat exerts on the ball? Ignore gravity in this problem.
The total momentum of an isolated system of objects is conserved, regardless of the nature of the force between the objects in that system.

Here, we speak of the linear momentum of the system.

Thus, if our system consists of two objects...

\[ p_{1i} = m_1 v_{1i} \]
\[ p_{2i} = m_2 v_{2i} \]

The total initial momentum of the system is... After these two balls hit, the situation changes...

\[ p_{sys f} = p_{1f} + p_{2f} = m_1 v_{1f} + m_2 v_{2f} \]

And by conservation of momentum...

\[ p_{sys f} = p_{sys i} \]
\[ p_{1f} + p_{2f} = p_{1i} + p_{2i} \]
\[ m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \]

Notice that this statement can also be derived through Newton’s Laws.

Let’s say the collision lasts for a time \( \Delta t \).

FBDs:
Newton's 3rd Law:

\[ m_1 \Delta \vec{p}_{1\text{tot}} = \vec{F}_{2\rightarrow 1} \Delta t \]
\[ m_2 \Delta \vec{p}_{2\text{tot}} = \vec{F}_{1\rightarrow 2} \Delta t \]

Newton's 3rd Law:

\[ \Delta \vec{p}_1 = -\vec{F}_{1\rightarrow 2} \Delta t \]
\[ \Delta \vec{p}_2 = \vec{F}_{2\rightarrow 1} \Delta t \]

\[ \Delta \vec{p}_1 = -\Delta \vec{p}_2 \]

Or stated slightly differently:

\[ \vec{p}_{1,f} - \vec{p}_{1,i} = -(\vec{p}_{2,f} - \vec{p}_{2,i}) \]
\[ \vec{p}_{1,f} + \vec{p}_{2,f} = -(\vec{p}_{1,i} + \vec{p}_{2,i}) \]

\[ \vec{p}_{\text{tot},f} = -(\vec{p}_{\text{tot},i}) \]

**Conservation of Momentum**

**Worksheet Problems #2**

**Concept Quiz!**

**CAUTION**

**WATCH YOUR STEP**