Physics 111

Lecture 12

Tuesday, October 05, 2004

- Ch 6: Last Example
- Ch 9: Impulse
  Momentum
Help this week:
Wednesday, 8 - 9 pm in NSC 118/119
Sunday, 6:30 - 8 pm in CCLIR 468
This week’s lab will be a workshop. Bring your Ranking Tasks workbook and your textbook.
Try the "elevator experiment"

Note the following:

How does your weight change while you’re…

• Ascending at $v = \text{const.}$?
• Descending at $v = \text{const.}$?
• Accelerating Down? ↓
• Accelerating Up? ↑
Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string passing over a frictionless pulley. The inclines are also frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string.
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For $m_1$

- $n_1$
- $T$
- $W_1$

For $m_2$

- $T$
- $n_2$
- $W_2$
Ch 6: Applying Newton’s Laws

Pictorial Representation:

Unknowns: \( T, a_{1,x1} \)

Knowns:

\[ \Delta x_1 = \Delta x_2 \quad v_{1,x1} = v_{2,x2} \quad a_{1,x1} = a_{2,x2} \]

\[ m_1 = 8.00 \text{ kg} \quad m_2 = 3.50 \text{ kg} \]

\[ \theta_1 = 35^\circ \quad \theta_2 = 35^\circ \]

\[ g = 9.81 \text{ m/s}^2 \]

\[ W_{1x} = m_1g \sin \theta_1 \quad W_{1y} = -m_1g \cos \theta_1 \]

\[ W_{2x} = -m_2g \sin \theta_2 \quad W_{2y} = -m_2g \cos \theta_2 \]

\[ a_{1,y1} = a_{2,y2} = 0 \]
Mathematical Representation:

Let’s look at Newton’s 2\textsuperscript{nd} Law in the $x$-direction for both blocks.

\[
\alpha_{1,x1} = \frac{F_{x1,\text{net}}}{m_1} = \frac{m_1 g \sin \theta_1 - T}{m_1}
\]

\[
\alpha_{2,x2} = \frac{F_{x2,\text{net}}}{m_2} = \frac{T - m_2 g \sin \theta_2}{m_2}
\]

These \textit{accelerations} must be the same!
Mathematical Representation:

\[
\frac{m_1 g \sin \theta_1 - T}{m_1} = \frac{T - m_2 g \sin \theta_2}{m_2}
\]

Rearranging and using the fact that \( \theta_1 = \theta_2 \ldots \)

\[
T \left( \frac{1}{m_1} + \frac{1}{m_2} \right) - 2g \sin \theta = 0
\]
Finally, plugging in numbers....

\[ T \left( \frac{1}{3.50 \text{kg}} + \frac{1}{8.00 \text{kg}} \right) - 2(9.81 \text{ m/s}^2) \sin 35^0 = 0 \]

\[ T = 27.4 \text{ N} \]

Now plugging back into Newton's 2nd Law we find

\[ a_{1,x1} = \frac{(3.50 \text{ kg})(9.81 \text{ m/s}^2) \sin 35^0 - 27.4 \text{ N}}{(3.50 \text{ kg})} = -2.20 \text{ m/s}^2 \]
In the study of systems with multiple objects, particularly systems in which these objects interact with one another through collisions, it is useful to take advantage of the concept of momentum. We’ll see that this new quantity is conserved in isolated systems, allowing us to analyze some rather complicated interactions.
We’ve applied Newton’s Laws to better understand the behavior of systems
- single object
- multiple object
- remote interactions

Now, let’s examine interactions between objects through another approach:
What constitutes an impulse is probably best illustrated through a simple demonstration.

When I drop the ball, the force of gravity exerts an impulse on the falling ball.

When the ball hits the floor, a second force also exerts an impulse: the contact force of the floor.
For a *constant force*, as with the falling ball, it’s easy!

\[ \vec{I} = \vec{F} \Delta t \]

Notice graphically that the **impulse** is the “area” under the curve of a **force vs time** graph.

(Just like **distance** was the “area” under a **velocity vs time** graph!)
Impulse

\[ \vec{I} = \vec{F} \Delta t \]

\[[\vec{I}] = [\vec{F}][\Delta t]\]

\[[\vec{I}] = \text{N s} = \frac{\text{kg m}}{\text{s}}\]
So, what about non-constant forces?

Let’s look at a simple case of a force that increases linearly with time.

The answer is still just the area under the curve.
So, what about non-constant forces?

Let’s assume that during the bounce, the floor exerts a force on the ball as given at left.

Note: The force of gravity results in an impulse at the same time.
A 0.200-kg rubber ball is released from a height of 2.00 m above the floor. The ball falls, bounces, and rebounds. How high does the ball make it on the rebound if the force function is given by the graph below?
This problem needs to be separated into 3 parts:
1) the ball falling from 2.00 m to the floor
2) the interaction of the ball with the floor
3) the ball rising to a new, unknown height

Motion Diagrams:

(1) 

(2) Close-Up 

(3)
Pictorial Representation (1):

**Knowns:**

\[ y_i = 2.00 \text{ m}, \quad y_f = 0 \]
\[ v_i = 0 \]
\[ t_i = 0 \]
\[ a = -g \]
\[ m = 0.200 \text{ kg} \]

**Unknowns:**

\[ t_f, \quad v_f \]
Mathematical Representation (1):

**Knowns:**

\[ y_i = 2.00 \text{ m}, \quad y_f = 0 \]
\[ v_i = 0 \]
\[ t_i = 0 \]
\[ a = -g \]
\[ m = 0.200 \text{ kg} \]

\[ 0 = 2.00m + 0 + \frac{1}{2}(-9.81 \text{ m/s}^2)(\Delta t)^2 \]

\[ \Delta t = t_f - t_i = t_f - 0 = t_f = 0.639 \text{ s} \]

**Unknowns:**

\[ t_f, \quad v_f \]

\[ v_{y,f} = v_{i,y} + a_y(\Delta t) \]

\[ v_{y,f} = 0 + (-9.81 \text{ m/s}^2)(0.639\text{s}) = -6.26 \text{ m/s} \]
Free-Body Diagram (2):

From the graph of force vs time and our definition of impulse, we can find an average force over the 10 ms of the bounce. We’ll base our FBD on this average force.

Mathematical Representation (2):

Newton’s 2nd law

\[ a_y = \frac{F_{y,net}}{m} = \frac{n - W}{m} = \frac{n - mg}{m} \]
Mathematical Representation (2):

To get the normal force, we need to use the graph and our definition of impulse.

\[ I = \text{area under curve} = \frac{(F_{\text{max}} + F_{\text{min}})}{2} (\Delta t) \]

\[ I = \frac{(480\,\text{N} - 0)}{2} (15\,\text{ms} - 5\,\text{ms}) = 2.4\,\text{N}\text{s} \]
Mathematical Representation (2):

To get the normal force, we need to use the graph and our definition of impulse.

\[
\langle \vec{F}_{\text{floor}} \rangle = \frac{\vec{I}_{\text{floor}}}{(\Delta t)} = \frac{+2.4 \text{Ns}}{10 \text{ms}} = 240 \text{N}
\]
Mathematical Representation (2):

Back to Newton’s 2\textsuperscript{nd} law…

\[ a_y = \frac{F_{y,net}}{m} = \frac{n - W}{m} = \frac{n - mg}{m} \]

\[ a_y = \frac{(240\,N) - (0.200\,kg)(9.81\,m/s^2)}{0.200\,kg} = 1190\,m/s^2 \]
Pictorial Representation (2):

**Knowns:**

- \( y_i = 0 \text{ m, } y_f = 0 \)
- \( v_i = -6.26 \text{ m/s} \)
- \( t_i = 0.639 \text{ s} \)
- \( t_f = 0.649 \text{ s} \)
- \( a_y = +1190 \text{ m/s}^2 \)
- \( m = 0.200 \text{ kg} \)

**Unknowns:**

- \( v_f \)

**Note:** The initial state of this part (2) of the problem was the final state of the previous part (1).
Mathematical Representation (2):

**Knowns:**

\[ y_i = 0 \text{ m}, \quad y_f = 0 \]
\[ t_i = 0.639 \text{ s} \]
\[ a_y = +1190 \text{ m/s}^2 \]

**Unknowns:**

\[ v_i = -6.26 \text{ m/s} \]
\[ t_f = 0.649 \text{ s} \]
\[ m = 0.200 \text{ kg} \]

\[ v_{y,f} = v_{i,y} + a_y (\Delta t) \]

\[ v_{y,f} = -6.26 \text{ m/s} + (1160 \text{ m/s}^2)(0.649 \text{ s} - 0.639 \text{ s}) \]

\[ v_{y,f} = 5.64 \text{ m/s} \]
Ch 9: Momentum & Collisions

Pictorial Representation (3):

**Knowns:**

\[ y_i = 2.00 \text{ m} \]
\[ v_i = +5.64 \text{ m/s} \]
\[ v_f = 0 \]
\[ t_i = 0.649 \text{ s} \]
\[ a = -g \]
\[ m = 0.200 \text{ kg} \]

**Unknowns:**

\[ t_f, y_f \]
Mathematical Representation (3):

**Knowns:**

\[ y_i = 2.00 \text{ m} \]
\[ v_i = +5.64 \text{ m/s} \]
\[ v_f = 0 \]
\[ t_i = 0.649 \text{ s} \]
\[ a = -g \]
\[ m = 0.200 \text{ kg} \]

\[ v_{y,f} = v_{i,y} + a_y(\Delta t) \]
\[ 0 = (5.64 \text{ m/s}) + (-9.81 \text{ m/s}^2)(t_f - 0.649 \text{ s}) \]

\[ t_f = 1.224 \text{ s} \]

\[ y_f = 0 + (5.64 \text{ m/s})(1.224 \text{ s} - 0.649 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(1.224 \text{ s} - 0.649 \text{ s})^2 \]

\[ y_f = 1.62 \text{ m} \]
We can always find the a constant $F$ such that the impulse delivered is the same as that delivered by a non-constant force.
The constant value is always exactly the time-averaged value of the non-constant force!

\[ \langle \vec{F}_{net} \rangle = \frac{\text{area under curve}}{\Delta t} \]
Having explored this quantity graphically and using results from calculus (not shown), we can say succinctly and generally that:

\[ \vec{I} = \left\langle \vec{F}_{\text{net}} \right\rangle \Delta t \]
The *linear momentum* of an object of mass $m$ moving with velocity $v$ is defined as the product of the mass and the velocity:

$$\vec{p} = m\vec{v}$$

- Notice that *momentum* is a *vector quantity*, which means that it must be specified with both a magnitude and direction.

- Also notice that the direction of the *momentum* is necessarily *parallel to the velocity*.
\[ \vec{p} = m \vec{v} \]

\[ [\vec{p}] = [m][\vec{v}] \]

\[ [\vec{p}] = \text{kg \ (m/s)} \]

OR

\[ [\vec{p}] = \text{N \ s} \]

The units suggest a relationship between force and momentum.
What happens when we apply a force to an object?

It accelerates.

Its velocity changes.

Its momentum changes.

The force imparts momentum.
In fact, we can relate force to momentum changes directly using Newton’s 2nd Law:

\[ \vec{F}_{\text{net}} = m\ddot{a} = m \frac{\Delta \vec{v}}{\Delta t} \]

This form is completely general, as it allows for changes in mass as well as velocity with time.