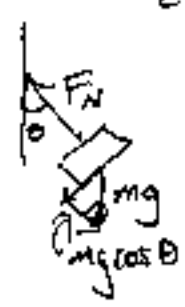


- 1) (a) Between points B and C, the normal force acts ~~opposite~~ <sup>toward</sup> the center of the small circle. Thus,  $F_N$  has a component that counters the force of gravity. Beyond point C,  $F_N$  has a component parallel to gravity. Therefore, the cart cannot leave the track before point C, but must leave the track between points C and D.

(b)  $E_{tot} = E_0 = mgR$  at point B,  $U = 0$ , so  $E_{tot} = \frac{1}{2}mv^2 = mgR$   
 $v^2 = 2gR$

(c)  $E_{tot} = mgR = \frac{1}{2}mv^2 + mgh$   
 $\frac{1}{2}v^2 = g(R-h) = g(R - \frac{R}{2}(1-\cos\theta)) = gR(1 - \frac{1}{2} + \frac{1}{2}\cos\theta)$   
 $v^2 = 2gR(\frac{1}{2} + \frac{1}{2}\cos\theta) = gR(1 + \cos\theta)$

(d) 

$$F_{net_c} = F_N - mg \cos \theta = \frac{mv^2}{r}$$

$$F_N = \frac{mv^2}{r} + mg \cos \theta$$

but  $r = R/2$  and  $v^2$  from (c)

$$F_N = 2mg(1 + \cos \theta) + mg \cos \theta$$

$$F_N = 2mg + 3mg \cos \theta$$

← clearly  $F_N \rightarrow 0$  before  $\theta = 180^\circ$  !!

When  $F_N = 0$ , the cart falls off the track

$$0 = 2mg + 3mg \cos \theta \leftarrow$$

$$\cos \theta = -2/3 \quad \theta = 131^\circ$$

- (e) Conservation of energy still holds because when the cart falls off the track, it has not only potential energy, but also kinetic energy. It is the sum of  $U + KE$  that remains constant!

2. For each of these questions, there is a conceptual way and a mathematical way to get to the right answer. I will describe both.

- a. Conceptually: We know from experience that the golf ball will bounce back from the bowling ball with roughly the same speed, just slightly slower. Therefore, the golf ball's momentum will be comparable in magnitude before and after the collision, about  $mv$ . Momentum is conserved, so the change in momentum of the golf ball will equal the change in momentum of the bowling ball. Since the momentum of the golf ball after the collision is in the opposite direction of the initial momentum, the change in momentum of the golf ball (and therefore the bowling ball's final momentum) has magnitude of  $2mv$ . Therefore, the bowling ball has more momentum.

Mathematically: Use the equations that result from conservation of kinetic energy and momentum in elastic collisions with object 1 (golf ball) initially moving at speed  $v$  and object 2 (bowling ball) at rest:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v$$

If  $m_2 \gg m_1$ , then

$$v_{1f} = -v$$

$$v_{2f} = \frac{2m_1}{m_2} v$$

and

$$|p_{1f}| = m_1 v$$

$$|p_{2f}| = m_2 v$$

so the momentum of the bowling ball is greater.

- b. Conceptually: The kinetic energy of the golf ball will decrease only slightly since the speed is about the same, whereas the bowling ball will have a tiny speed. So, even though mass is a factor in kinetic energy, the squared speed rules the day.

Mathematically: The kinetic energy of the golf ball after the collision will be approximately  $KE_g = \frac{1}{2} m_1 v^2$ . The KE of the bowling ball after collision (using previous result) will be

$$KE_B = \frac{1}{2} m_2 v_{2f}^2 = \frac{2m_1^2}{m_2} v^2$$

Since

$$\frac{1}{2} > \frac{2m_1}{m_2}$$

the kinetic energy of the golf ball is more.

- c. Conceptually: From the point of view of the bowling ball (or in fancy terms, reference frame), the collision looks the same as in the original case. So there should be the same momentum transfer, and so the golf ball should have a speed of  $2mv$ . The bowling ball's velocity won't change much at all, but the golf ball will bounce off of it like a tennis ball during a serve.

Mathematically: See Conceptual Checkpoint 9-4 in Walker. Using

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v$$

again, but this time object 1 is bowling ball and object 2 is golf ball, we find in the case that  $m_2 \ll m_1$  that

$$v_{1f} = v$$

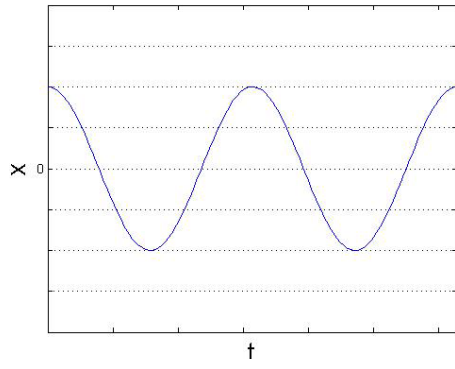
$$v_{2f} = 2v$$

- d. Conceptually: There will be almost no change of momentum of the bowling ball. The final momentum of the golf ball will be small compared to the bowling ball, even though it will be moving twice as fast, because it has such a smaller mass.

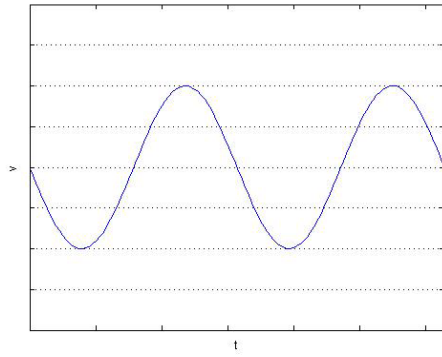
Mathematically:  $m_1 v > 2m_2 v$ .

- e. Conceptually: Although the golf ball is moving twice as fast, the mass of the bowling ball makes up for this fourfold factor in the kinetic energies. The bowling ball has more KE.

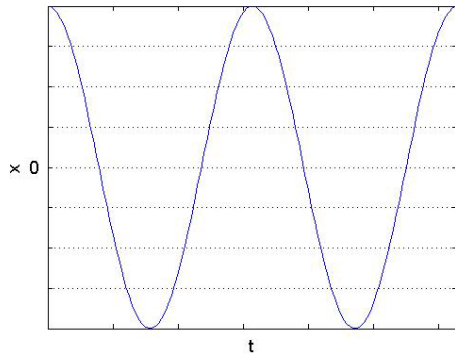
Mathematically:  $\frac{1}{2} m_1 v^2 > 2m_2 v^2$



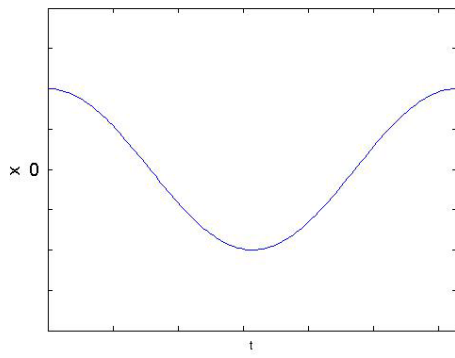
a.



b.



c.



d.

